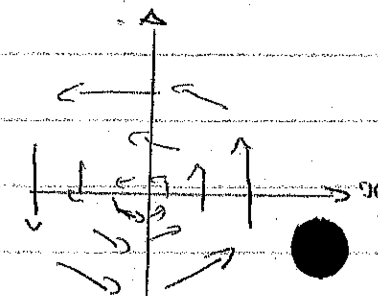
Vector Field: How to test if a Field is conservative / how to Find the potential.

If $\vec{F} = \vec{\nabla} f$ $\quad M = f_x$, $N = f_y$ $\Big\}$ then $f_{xy} = f_{yx}$ $\Rightarrow M_y = N_x$.

$\vec{F} = \langle M, N \rangle$

So if $\vec{F} = \langle M, N \rangle$ is defined and differentiable everywhere and if $M_y = N_x$ then $\vec{F}$ is a gradient Field.

<u>Example</u> $\vec{F} = -y\vec{i} + x\vec{j}$



is it conservative?

$M_y = -1 \neq N_x = 1$

so $\vec{F}$ is not a gradient Field. A magnetic Field is not a gradient Field you can get energy on a close path.

<u>Example</u> $\vec{F} = (4x^2 + axy)\vec{i} + (3y^2 + 4x^2)\vec{j}$

Find the value of a, For which $\vec{F}$ is conservative.

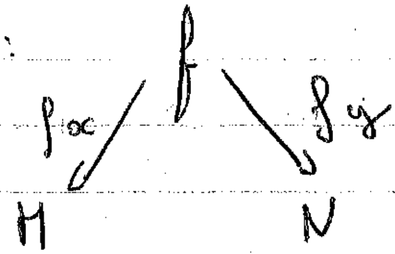
$M_y = ax \quad N_x = 8x \Rightarrow a = 8$

Suppose $\vec{F} = (4x^2 + 8xy)\vec{i} + (3y^2 + 4x^2)\vec{j}$

Find the potential $\vec{\nabla} F = f(x, y)$

2 ways to do it

① use antiderivatives:



$f_x = 4x^2 + 8xy$ integrate

$f(x,y) = \frac{4x^3}{3} + \frac{8x^2}{2}y + C(y) = \frac{4x^3}{3} + 4x^2y + C(y)$

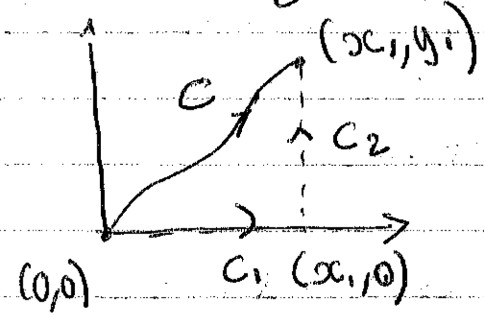
also $f_y = N \Rightarrow$ derive f with respect to y

$f_y = \cancel{4x^2} + C'(y) = 3y^2 + \cancel{4x^2}$

$C'(y) = 3y^2 \Rightarrow C(y) = y^3 + C$

so $f(x,y) = \frac{4x^3}{3} + 4x^2y + y^3 + C$

② 2nd way use line integral.



constant

consider $\int_C \vec{F} \cdot d\vec{r} = f(x_1, y_1) - f(0,0)$

we need to find $\int_C \vec{F} \cdot d\vec{r}$ to find $f(x,y)$

~~instead~~ instead of using $\int_C$ we

use $\int_{C_1} + \int_{C_2}$. along C_1 $dy = 0$ and $y = 0$
along C_2 $dx = 0$ and $x = x_1$

$$\int_C \vec{F}.d\vec{r} = \int_{C_1} (4x^2 + 8xy)\,dx + \int_{C_2} (3y^2 + 4x^2)\,dy$$

$$= \int_{(0,0)}^{(x_1,0)} (4x^2 + 8xy)\,dx + \int_{x_1,0}^{(x_1,y_1)} 3y^2 + 4x^2\,dy$$

$$\left[\frac{4x^3}{3} + \frac{8x^2y}{2}\right]_0^{x_1} + \left[\frac{3y^3}{3} + 4x^2y\right]_{x_1,0}^{x_1,y_1}$$

$$= \frac{4x_1^3}{3} + \frac{3y^3}{3} + 4x^2y$$

$$f(x,y) = \frac{4x^3}{3} + y^3 + 4x^2y + f(0,0).$$

we get the same thing.

New notation, close path is noted: $\oint_C$

So if $\vec{F}$ is conservative $\oint_C \vec{F}.d\vec{r} = 0. \Leftrightarrow M_y = N_x.$