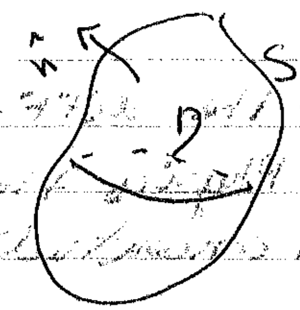
II

<u>divergence theorem _ Gauss-Green theorem.</u>



S is a close surface enclosing a region D oriented with $\hat{n}$ outward

$\vec{F}$ defined and differentiable everywhere in D, then:

$$\oiint_S \vec{F}\cdot d\vec{s} = \iiint_D \operatorname{div}\vec{F}\,.dV.$$

where $\operatorname{div}(\vec{F}) = F_{1x} + F_{2y} + F_{3z}$.

<u>Example 1</u> Flux $z\vec{k}$ through a sphere a radius a

$$\oiint_S z\vec{k}\cdot\vec{n}\,ds = \iiint_D \underbrace{\operatorname{div}(z\vec{k})}_{1}\,dV = \frac{4}{3}\pi a^3$$

<u>Diffusion equation.</u>

this equation governs diffusion of smoke in air or temperature or dye in a solution.

The unknown function is $u(x,y,z,t)$

concentration changes with time. $\frac{\partial u}{\partial t}$

$$\frac{\partial u}{\partial t} = K\nabla^2 u = K\vec{\nabla}\cdot\vec{\nabla}u = K\operatorname{div}(\vec{\nabla}u)$$

$\hookrightarrow$ laplacian

$$\boxed{\frac{\partial u}{\partial t} = K\operatorname{div}(\vec{\nabla}u)}$$

proof: $\vec{F}$ is the vector field associated could be the velocity of flux.

$\vec{F}$ = flow of smoke.

(1) the flow moves faster if the difference is density is larger. In Physics the flows goes from high concentration to low concentration.

$$\boxed{\vec{F} = -K \vec{\nabla} u.}$$

(2) we use the divergence theorem.

$$\underbrace{\operatorname{div} \vec{F}}_{} = -\frac{\partial u}{\partial t}.$$

the amount of fluid leaving per unit volume = Δu

per unit time $= -\frac{\partial u}{\partial t}$

negative sign because if density ↑ the flux is the other way.

$$\frac{\partial u}{\partial t} = -\operatorname{div} \vec{F} = +K \operatorname{div}(\vec{\nabla} u)$$

$$\boxed{\frac{\partial u}{\partial t} = K \nabla^2 u}$$