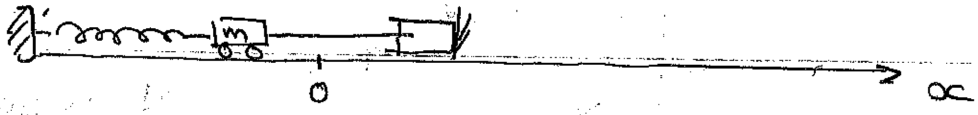
More on oscillations - $y'' + Ay' + By = 0$

complex roots - the system is underdamped.



$$\underbrace{m\ddot{x}}_{} + \underbrace{C\dot{x}}_{\text{damping}} + \underbrace{Kx}_{\text{hookes Law}} = 0 \Rightarrow \ddot{x} + \frac{C}{m}\dot{x} + \frac{K}{m}x = 0$$

Let's rewrite the equations as: $y'' + 2py' + \omega_0^2 = 0$.

(will be easier with the quadratic Formula)

quadratic equation: $r_\pm = \dfrac{-2p \pm \sqrt{4p^2 - 4\omega_0^2}}{2}$

$r = -p \pm \sqrt{p^2 - \omega_0^2}$ $\quad \left(2p = \dfrac{C}{M} \text{ and } \omega_0^2 = \dfrac{K}{m}\right)$

special case if $p = 0$, $C = 0$, there is no damping.

the motion is harmonic. $r = \pm \omega_0$.

$y = A\cos\omega_0 t + B\sin\omega_0 t$ with $\omega_0 = \sqrt{\dfrac{K}{m}}$

A and B are given by initial conditions.

or $\boxed{y = C\cos(\omega_0 t - \phi)}$

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if $p^2 - \omega_0^2 < 0$ $\quad p < \omega_0$ $\quad (C^2 < 4MK)$

you get damped oscillations.

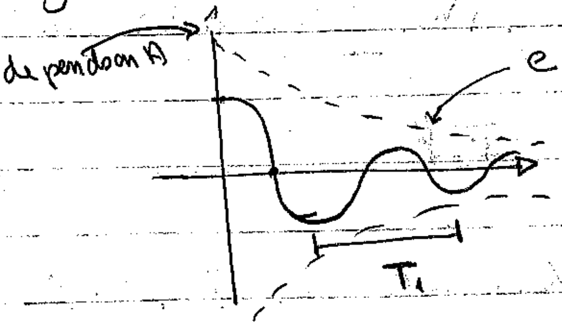
$r = -p \pm \sqrt{-(\omega_0^2 - p^2)} = -p \pm i\sqrt{\omega_0^2 - p^2}$

$= -p \pm i\sqrt{\omega_1^2}$

Solution $y = e^{-pt}(C_1\cos\omega_1 t + C_2\sin\omega_1 t)$

$\omega_1^2 = \omega_0^2 - p^2$

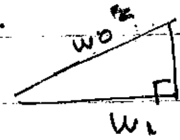
$$y = e^{-pt} A \cos(\omega_1 t - \phi)$$



$\omega_1 = \frac{2\pi}{T_1}$

T_1 is called the ~~spe~~ pseudo-period.

$\omega_1^2 = \omega_0^2 - p^2$



$p \rightarrow$ depends on damping (system)

$\left.\begin{matrix} A \\ \phi \end{matrix}\right\}$ depends on the initial conditions

$\omega_1 \rightarrow$ depends on spring and damping (system)

We need 4 parameters to describe the system.

e^{-pt} ↳ rules how fast it is coming down.

A ↳ amplitude of exponential.

ϕ ↳ phase lag.