2nd approach to find the solution of $y'' + Ay' + By = 0$ when the roots are complex (oscillations)

$r_{\pm} = a \pm bi$ (complex roots)

instead of: $y = \underbrace{e^{at}\cos bt}_{\text{real part}} + \underbrace{e^{at}\sin bt}_{\text{Imaginary part.}}$

we use a complexe solution:

$$\tilde{y} = C_1 e^{(a+bi)t} + C_2 e^{(a-bi)t}$$ with C_1 and C_2 complex numbers.

But we need to find conditions on C_1 and C_2 such as $\tilde{y}$ is real. That would mean Find C_1 and C_2 such as $\tilde{y}$ is real. In that case if we change i to $-i$ $\tilde{y}$ is unchanged.

$$C_1 e^{(a+bi)t} + C_2 e^{(a-bi)t} = \overline{C}_1 e^{(a-bi)t} + \overline{C}_2 e^{(a+bi)t}$$

$\Rightarrow$ $C_1 = \overline{C}_2$ or $C_2 = \overline{C}_1$ they are conjugate of eache other.

So $y = C e^{(a+bi)t} + \overline{C} e^{(a+bi)t}$

or

$$y = (c+id)e^{(a+bi)t} + (c-id)e^{(a-bi)t}$$

$$y = e^{at}\left[c\left(e^{bit} + e^{-bit}\right) + id\left(e^{bit} - e^{-bit}\right)\right]$$

$$\boxed{y = e^{at}\left[2c\cos bt - 2d\sin bt\right]}$$

Same Thing as before.