Linear, 1st order differential equation. cont'

Let's consider again:

$$y' + k_2 y = k_2 \cos \omega t \quad (y' + ky = kq(t) \text{ type})$$

In the complex domain, the equation becomes:

$$\tilde{y}' + k \tilde{y} = k_2 e^{i\omega t}$$

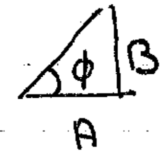
the solution is $\frac{1}{1 + i\omega/k_2} e^{i\omega t} = \tilde{y}$ $\Big\}$ $y = \mathrm{Re}(\tilde{y})$

and we found the solution by going polar and by finding the real part of $\tilde{y}$.
we can also find y by going cartesian:

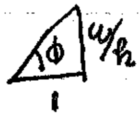
$$\tilde{y} = \frac{1}{1 + i\omega/k} e^{i\omega t} = \frac{1 - i\omega/k}{1 + \omega^2/k^2} e^{i\omega t} = \frac{1 - i\omega/k}{1 + \omega^2/k^2} (\cos \omega t + i \sin \omega t)$$

$$\mathrm{Re}(\tilde{y}) = \frac{1}{1 + \omega^2/k^2} \left(\cos \omega t + \frac{\omega}{k_2} \sin \omega t\right)$$

$\Big\{$ we know: $A \cos \theta + B \sin \theta = C \cos(\theta - \phi)$
with $C = \sqrt{A^2 + B^2}$ and $\tan \phi = B/A$



$$\cos \omega t + \frac{\omega}{k} \sin \omega t = \sqrt{1 + \frac{\omega^2}{k^2}} \cos(\omega t - \phi) \text{ with}$$

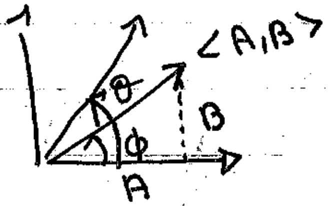


$$\boxed{y = \frac{1}{\sqrt{1 + \omega^2/k^2}} \cos(\omega t - \phi) \text{ with } \tan \phi = \omega/k}$$

We find the same solution.

* proof of $A\cos\theta + B\sin\theta = C\cos(\theta-\phi)$

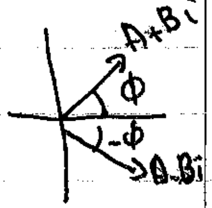
proof 1 consider the vector $\langle A,B\rangle$ and $\langle \cos\theta, \sin\theta\rangle$



$$A\cos\theta + B\sin\theta = \overbrace{\langle A,B\rangle \cdot \langle \cos\theta, \sin\theta\rangle}^{\text{dot product}}$$

$$= \sqrt{A^2+B^2}\,(1)\cos(\theta-\phi)$$

proof 2



$$A\cos\theta + B\sin\theta = \mathrm{Re}\big((A-Bi)(e^{i\theta})\big)$$

$$= \mathrm{Re}\, e^{\sqrt{A^2+B^2}} e^{-\phi i} e^{i\theta} = \cancel{\mathrm{Re}\, e^{(\theta-\phi)i} = \cos(\theta-\phi)}$$

$$= \mathrm{Re}\left(\sqrt{A^2+B^2}\; e^{(\theta-\phi)i}\right) = \sqrt{A^2+B^2}\cos(\theta-\phi)$$