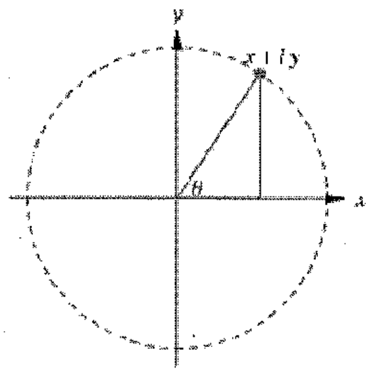
REVIEW complex number Type equation here.**s / for solving differential equations**$\sqrt{\ }$

A complex number is written as: z = a + bi a is the real part and bi the imaginary part. $i = \sqrt{-1}$

The modulus is r = $\sqrt{(a^2 + b^2)}$, the argument is ∅ with tan(∅) = b/a

or z = r (cos(∅) + i sin (∅))

A complex number can be represented as a vector on a circle :



The x-component x = a = r cos (∅) and the y-component is y = b = r sin (∅)

$\bar{z}$ = a – bi   is the conjugate.

The polar representation is very useful: z = r $e^{i\Phi}$

* we can easily find z1z2 = r1 r2 $e^{i(\Phi1+\Phi2)}$

argument(z1z2)= $\Phi1 + \Phi2$, modulus(z1z2) = r1 r2

* we can easily find 1/z = 1/r $e^{-i\Phi}$ 1/r = $\frac{1}{\sqrt{(a^2+b^2)}}$, $\tan(\Phi) = \frac{b}{a}$

* we can find the nth root of unity. Let's find the third root of 1 in the complex number system:

$\sqrt[3]{1}$ = z or 1= z^3 or 1 = r $e^{i3\Phi}$ with r = 1 if $e^{i3\Phi}$ = 1 then 3 Φ = 2л or 3 Φ = 4л or 3 Φ = 6л

so $\Phi1$ = 2 л/3 or $\Phi2$ = 4 л/3 or $\Phi3$ = 2 л The 3 solutions are placed at the 3 vertices of an equilateral triangle. So you have 3 solutions.

z1 = cos(2 л/3) + i sin (2 л/3) , z2 = cos(4 л/3) + i sin(4 л/3) , z3 = 1

*you can use the polar representation to prove : A $\cos(\Phi) + B\sin(\Phi)$ = C cos ($\Phi - \Theta$)
with C = $\sqrt{A^2 + B^2}$ and tan (Θ) = B/A

proof: Acos (Φ) = Re (A$e^{i\Phi}$) and Bsin(Φ) $= Re(-Bie^{i\Phi})$

A $\cos(\Phi) + B\sin(\Phi)$ = Re(A$e^{i\Phi}$ $- Bie^{i\Phi}$) $= Re$ ($e^{i\Phi}$ (A – Bi)) = Re ($\sqrt{A^2 + B^2}\, e^{i(\Phi-\Theta)}$)

=$\sqrt{A^2 + B^2}$ cos ($\Phi - \Theta$) with tan (Θ) = B/A

* Like wise you can use the polar representation to prove:

10cos(wt) + 5 cos(wt + pi/2) = $\sqrt{A^2 + B^2}$ cos (wt + Θ) with tan(Θ) = B/A

Sum = Re (10 e^{iwt} + 5 $e^{(iwt + pi/2)}$) = Re(e^{iwt} (10 + 5 $e^{ipi/2}$) = Re (e^{iwt} (10 + 5i)

= Re ($\sqrt{A^2 + B^2}\, e^{i(wt+\Theta)}$) with tan ($\Theta$)= 5/10

= $\sqrt{A^2 + B^2}\cos(wt + \Theta)$

NOTE : you can also use the Fresnel representation. See: http://cloverde.free.fr/devoirs/fresnel/cfresnel.htm

It is even easier.

* you can use the polar representation to find integral like :

$\int e^{-x} \cos x\, dx$ The integral becomes Re($\int e^{-x} e^{ix} dx$)

and $\int e^{-x} e^{ix}\, dx = \int e^{x(i-1)}\, dx = \frac{1}{i-1} e^{x(i-1)} = \frac{i+1}{2}\, e^{x(i-1)}$ The real part is **e^{-x}/2 (cos – sinx)**